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Three-dimensional effects in wind tunnel studies of shock wave reflection

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This paper concentrates on establishing the three-dimensional flow geometry associated with studies of shock wave reflection between two symmetrical wedges in supersonic flow. It considers the issue of hysteresis in such flows, and draws a distinction between three different aspects of hysteresis, associated with: ideal two-dimensional flow, flow with noise, and three-dimensional effects. The three-dimensional nature of the flow field is elucidated by the use of oblique shadowgraph photography where the optical axis of the shadowgraph system passes at an oblique angle, of as much as 55°, through the test section. The traces of the wave system reflecting off the tunnel window are identified and are used to assist in identification of wave profiles. The nature of the approach of the peripheral Mach reflections collapsing towards the centre of the flow becomes evident, as does the mechanism of transition from Mach reflection to regular reflection. Distinct evidence of the effects of flow perturbations at the mechanical equilibrium transition point are presented, as are changes in the rate of growth of the Mach stem near this point.

It is shown that three-dimensional effects can have a major effect on the wedge angle for transition. In the present tests, at Mach 3.1 and a wedge aspect ratio of 0.5, this occurs at a wedge angle of about 5° higher than the theoretical maximum for the corresponding two-dimensional flow, where the dual solution domain extends over only two degrees.

1. Introduction

Consider the supersonic flow between two finite aspect ratio wedges symmetrically situated about a horizontal plane in a supersonic wind tunnel. This experimental set-up is the conventional means of exploring shock wave reflection phenomena in steady flows without the confounding influence of surface boundary layer effects, the plane of symmetry acting as an ideal, rigid, adiabatic, and perfectly smooth surface. Finite span wedges are used in order to avoid interaction effects between the bow shock and wedge, with the boundary layers on the side (window) walls. It has been assumed until very recently that if the wedges are of a large enough span in comparison to their separation that the flow between them, as imaged by schlieren or shadowgraph photography, will be two-dimensional, and therefore will allow legitimate comparison with two-dimensional oblique shock theory. Whilst the plane of symmetry does not have to be horizontal, this terminology will be used throughout this paper in the interests of clarity. Thus when reference is made to the vertical plane of symmetry this refers to a plane passing through the mid-section of both wedges.

Central to the studies has been the question of transition from regular to Mach

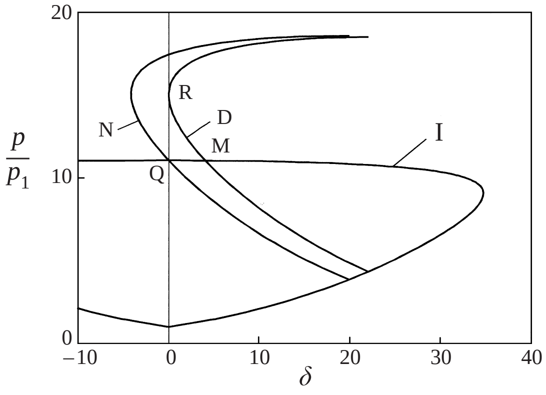

FIGURE 1. Shock polars for $M = 3.1$. The dual solution domain is between reflected shock polars N and D.

reflection, and back, as the angle of the wedges change for a given flow Mach number. Both numerical and experimental studies have been undertaken and compared with von Neumann's well-known two-shock theory (von Neumann 1943). This theory shows that there is an incident shock angle beyond which there is no reflected shock that can turn the flow back parallel to the initial flow direction. This is known as the extreme, or detachment, angle. For weak shocks and angles larger than this limit the reflection transits to Mach reflection where the incident and reflected shocks meet at a point away from the plane of symmetry (the triple point), and the triple points on either side of the plane of symmetry are connected by a third shock, called the Mach stem. The major area of interest, however, occurs for strong shocks, in an operating region where both regular and Mach reflection are theoretically possible. In this context strong shocks will be taken as being those where this dual solution domain exists, which for a specific heat ratio of $\gamma = 1.4$ corresponds to free-stream Mach numbers greater than 2.23.

The issue can be usefully illustrated using pressure–deflection shock polars as shown in figure 1. The polar marked I represents the locus of all possible states in the pressure–deflection plane which can be reached by a plane shock wave in a flow with a fixed upstream Mach number ($M = 3.1$ for the case shown). Two reflected shock polars are shown which show all states that can be achieved through a reflected shock for a given state behind the incident shock, for two different deflection (wedge) angles. Curve D represents the upper limit of the dual solution domain. Here the reflected shock at R is just able to bring the flow back to zero deflection and thus corresponds to the detachment condition. A slightly higher initial deflection through the incident shock resulting from a slightly larger wedge angle would result in the reflected shock polar not intersecting the pressure axis and regular reflection would not be possible. The polar D intersects the incident polar at M. This is the condition for the corresponding Mach reflection with the pressure and overall deflection behind the oblique reflected shock (on the reflected polar) being the same as that immediately behind the single Mach stem (on the incident polar). Note however, that the pressure behind the regular reflection solution is very much higher than that for the Mach reflection solution. The polar marked N represents the lower limit of the dual solution domain. Here the Mach reflection solution and the regular reflection solution both occur at point Q. Thus both types of reflection result in the same downstream pressure. It is for this reason that Henderson & Lozzi (1975) labelled this condition the mechanical equilibrium condition, since it is the only situation where there is a smooth transition of pressure, from the one type of reflection to the other as the wedge changes angle, with the detachment condition being considered aphysical due to the required sudden jump in pressure at transition. This equilibrium condition is frequently referred to as the von Neumann condition in the literature, since he originally proposed it.

Hornung, Oertel & Sandeman (1979) argued that transition must be linked to some physical length scale since the length of the Mach stem is determined by the scale of the experiment. Thus for Mach reflection to occur information concerning the physical scale of the experiment must be communicated to the reflection point. Information on the length of the wedge is communicated through the expansion fan springing from the trailing edge of the wedge to the subsonic patch behind the Mach stem. This led these authors to conclude that, for increasing wedge angle, regular reflection is maintainable, through the von Neumann point, until the sonic condition is reached, whereas with decreasing angle Mach reflection should occur all the way down to the von Neumann point below which it is no longer possible, and hysteresis in the reflection pattern would be expected. A similar argument regarding the opening of information pathways will be presented below, but in this case relating to transverse signals.

Wedge angles with shock polars starting to the right of case D will always result in Mach reflection, whereas those starting to the left of curve N will always give regular reflection, provided the flow is free of downstream influences. There is one other special important case which needs to be considered, and that is where the solution to regular reflection results in a sonic flow immediately downstream of the reflected shock. This case, the sonic condition, occurs on a polar very close to that for detachment conditions; in the Mach 3.1 example shown in figure 1 the sonic condition wedge angle is only 0.15° below the detachment value. Because of this small difference the distinction between it and the detachment condition is seldom made. It is common in the literature in discussing the bounds of hysteresis to set these as being between the von Neumann condition and detachment, rather than between the von Neumann point and the sonic condition as was originally proposed by Hornung *et al.* (1979). In the current discussion the sonic condition will similarly be identified as being of major importance, in this case in establishing whether transverse disturbances can influence the reflection.

The question that then arises within the dual solution domain, between polars N and D, is which type of reflection actually occurs, and under what conditions. Thus in moving from regular reflection towards transition, regular reflection would be maintained until it is no longer possible, i.e. at detachment. Therefore, starting with regular reflection, as the wedge angle is increased the operating point (i.e. the conditions behind the reflected shock) would move up the pressure axis of the polar with $\delta = 0$ until point R was reached and then the reflection would jump to point M, and would then proceed towards the right on the incident polar as the angle increased further. When decreasing the wedge angle, however, from the Mach reflection domain, the reflection could stay in this format at an operating point all the way along the incident shock polar to Q, beyond which only regular reflection would occur.

The main initial experimental studies (Henderson & Lozzi 1975, 1979, at $M = 2.95$; Hornung & Kychakoff 1977, at $M = 16$; and Hornung *et al.* 1979) all showed that transition occurred at the von Neumann condition, i.e. at point Q. Additional tests specifically designed to check the hysteresis postulate at flow Mach numbers of 2.48 and 4.96 (Hornung & Robinson 1982) as well as an extensive set of results with large aspect ratio wedge pairs (Fomin *et al.* 1996) at Mach numbers of 4, 5, and 6 confirmed the result that transition occurs in the vicinity of the mechanical equilibrium point irrespective of from which direction the wedge angles were changed.

The lack of significant hysteresis has led to suggestions that this could be due to regular reflection being less stable than Mach reflection in the dual solution domain (Hornung 1997) and that perturbations in the flow were causing the trigger to Mach reflection. The thorough experimental study by Fomin *et al.* (1996) has indeed shown that the small-scale hysteresis is dependent on the wind tunnel facility. Both free jet and closed test section tunnels of various sizes were tested, with the open jet facility giving a slight hysteresis, assumed by the authors to be due to lower levels of disturbance. This minor hysteresis was nowhere near the extent of the angle difference between the von Neumann and detachment points. Thus it was concluded that unavoidable disturbances existing in a steady-flow wind tunnel are sufficient to trip the flow to the more stable Mach reflection. It is interesting to note that two-dimensional numerical studies (Ivanov, Gimelshein & Beylich 1995; Ivanov *et al.* 1996; Chpoun *et al.* 1995) do indeed show the full extent of the hysteresis suggested by Hornung, extending from von Neumann to detachment conditions. Furthermore, some numerical studies have shown that by perturbing the flow the transition can be manipulated, thereby confirming that flow quality is indeed a factor in the transition process.

The detailed studies (Fomin *et al.* 1996) also showed that greater hysteresis occurred as the wedge aspect ratio was decreased below 2.5. The one earlier study in which significant hysteresis had been reported (Chpoun *et al.* 1995), and which had been treated as being two-dimensional, was done with an aspect ratio significantly below the values determined from the earlier work of Henderson and of Hornung referred to above, and is now generally accepted to have been strongly influenced by three-dimensional effects. Both numerical and experimental work (Ivanov *et al.* 1997*b*) have also clearly shown the significant effects on Mach stem height as the aspect ratio is changed. Thus the Mach stem heights are reduced by more than a quarter, for a incidence angle of 44°, $M = 4$, in going from a two-dimensional (infinite aspect ratio) numerical calculation to a three-dimensional calculation with an aspect ratio of 3.75. It should be noted that the latter calculation has been confirmed by experiment. Whilst the difference between the transition angle for these two cases was found to be negligible, the above difference does illustrate the strong influence that three-dimensional effects can have on the flow. Previously an experiment with an aspect ratio of 3.75 would have been regarded as an essentially two-dimensional flow. Thus it is not enough to assume that an experimental flow is two-dimensional if the wedge angles for transition coincide.

Consideration has recently been given to the likely geometry of the flow due to the presence of the edge of a finite aspect ratio wedge in a wind tunnel (Skews, Vukovic & Draxl 1996). This arose from some unexpected features noted near transition in doing some dynamic tests using a narrow wedge (Scott & Skews 1996). The method used (Skews 1997) to establish the influence of the edge is to calculate the minimum aspect ratio necessary to prevent the Mach cone between the incident shock and the wedge surface, and which arises from the flow perturbations due to the wedge edge, from intercepting the reflection point on the tunnel centreline. It shows, for example at Mach 5, that an inlet aspect ratio (wedge leading edge separation/wedge span) of at least 0.67 is required to avoid edge interference at the reflection point. This value is very similar to that of the experiments of Chpoun *et al.* (1995) which showed significant hysteresis, and confirmed that these data were influenced by edge effects. However, the experiments of Fomin *et al.* (1996) showed aspect ratio influences at inlet aspect ratios even larger than these theoretical minima indicated. The flow geometry considered by Skews (1997) showed that the perturbations carried from the edge within the Mach cone between the wedge surface and the incident shock would intersect with the reflected shock, and these edge perturbations would then be transmitted to the flow behind the reflected shock. This would generate a secondary Mach cone in this latter region. Since the Mach number of the flow behind the reflected shock is less than that behind the incident shock, this Mach cone has a larger Mach angle and its influence would penetrate markedly into the steady core flow behind the reflected shock. The important point is that when this flow becomes sonic the whole of the back face of the reflected shock becomes accessible to these edge signals. For increasing wedge angle this occurs before the detachment condition is reached. Thus in tests with decreasing wedge angle the reflected shock is accessible to these transverse signals when it passes through the detachment condition. This suggests, strictly speaking, that it is not appropriate to do wind tunnel tests using finite aspect ratio wedges to determine two-dimensional transition conditions with decreasing wedge angle, or (as noted above for the 3.75 aspect ratio tests) for two-dimensional Mach reflection studies even if these are at angles below the sonic condition, since again there are regions of subsonic flow downstream of the reflection point, which is then accessible to transverse influence.

The above work showing that there is a transverse information path has led to the conclusion that the aspect ratio required for the effective elimination of three-dimensional effects lies far beyond that which was previously assumed, particularly for transition studies from regular to Mach reflection, and that transition effects for small aspect ratios should not be analysed in terms of two-dimensional theory as done in Chpoun *et al.* (1995).

A further issue of major importance that has also been identified (Skews 1997) is the encroachment of a far-field or peripheral Mach reflection towards the tunnel centreline as the wedge angle is increased. For the case of two axisymmetric (Sears–Haack) bodies in close proximity Marconi (1983) has shown that whilst the bow wave reflection immediately between the bodies is regular, as the line of reflection moves away from the central plane, transversely and downstream, a stage will be reached where the reflection will undergo transition to Mach reflection. The reason for this behaviour becomes apparent by considering successive downstream cross-sections of the flow normal to the flow direction. The two bow waves are then initially circular and go through transition from normal reflection through regular reflection to Mach reflection as the angle of incidence between the waves increases, in exactly the same way (by analogy) that cylindrical blast waves reflecting off a surface change from regular to Mach reflection as they propagate outwards.

The studies summarized above are leading to an increasing interest in the true nature of these three-dimensional flows. A most interesting study in this field has been the recent three-dimensional numerical simulations of Ivanov *et al.* (1997*a*). These show that the simple monotonic decrease in Mach stem height from the periphery towards the centre when the reflection in the horizontal symmetry plane is Mach reflection throughout, as surmised by Skews (1997) in his conceptual visualization of the flow, is not borne out by the calculations. These show that the stem height decreases from that on the vertical plane of symmetry and then increases again further out. As the wedge angle is decreased it is shown that a situation can arise where there is Mach reflection on the centreline, followed by regular reflection further out and then by Mach reflection again at the periphery of the flow. These calculations confirm the divergence of the streamlines on the horizontal plane of symmetry, and show that the peripheral Mach surface is strongly convex. Note also that as this Mach surface

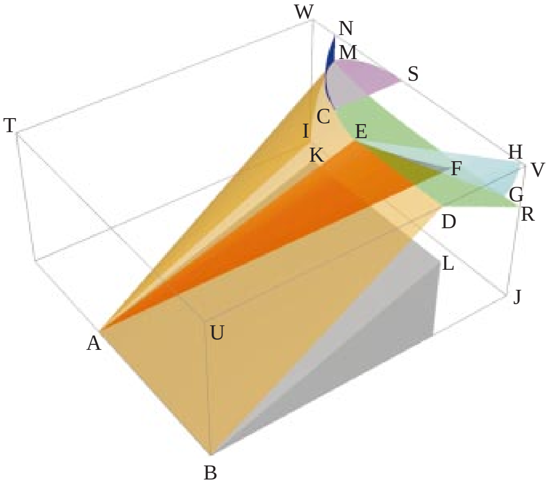

FIGURE 2. Influence of an edge on supersonic flow over a wedge.

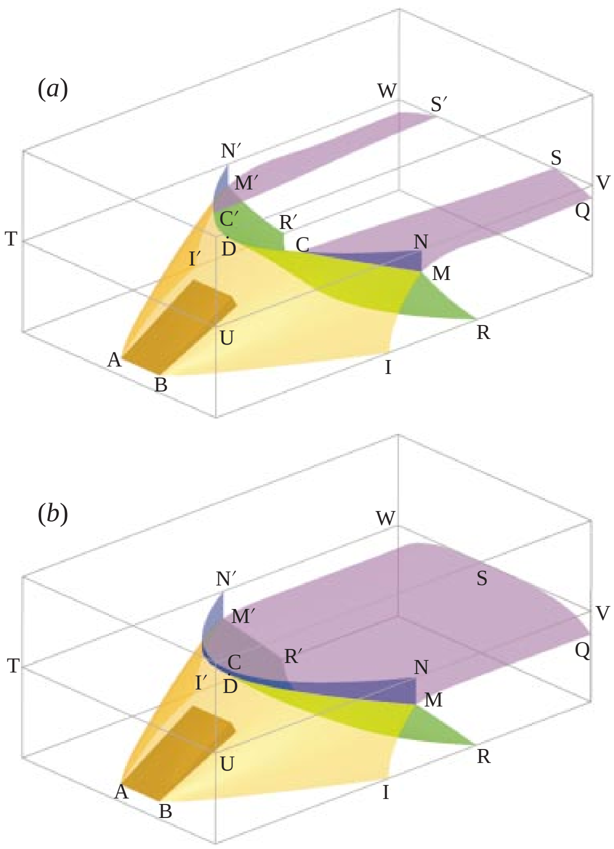

FIGURE 4. Schematic representation of waves and surfaces in three-dimensional flows: (a) regular reflection and (b) Mach reflection, in the vertical plane.

sweeps back the flow will remain supersonic on passing through the shock because of it being oblique to the flow. However, as shown in Ivanov *et al.*'s calculations (1997*a*) there is a subsonic patch in the vicinity of the tunnel centreline in which the flow is non-uniform. These calculations show that the streamlines diverge in the horizontal plane, whereas it is also known that for Mach reflection they converge in the vertical plane due to the upper and lower slipstreams approaching each other.

The above discussion suggests that there are three aspects relating to the appearance and extent of hysteresis: (*a*) that initially proposed by Hornung for ideal two-dimensional steady flows, which will be referred to as Hornung hysteresis, and which has been demonstrated through ideal two-dimensional numerical simulations and which extends from the von Neumann to the sonic condition; (*b*) that due to noise, which has been demonstrated by artificial perturbations in such calculations and has to do with the relative stability of the flows (see Hornung 1997), and also by comparative experiments under the same conditions in different wind tunnels, where it occurs in the region of the von Neumann point; and (*c*) that due to three-dimensional effects. Comprehensive experimental and numerical studies for this last case are still awaited. There is not yet a theoretical framework available to suggest what the extent of the dual solution domain for these flows may be. This paper addresses some aspects in this area, and, in particular, concerns itself with experimental visualization of some of the three-dimensional features which have not previously been explored. It concentrates on the geometry of the shock wave reflection transverse to the main flow.

2. Background on edge effects

The essentials of the edge-effect geometry are given in figure 2 for the case where the flow on the tunnel centreline, and the regular shock reflection there, is a two-dimensional flow because it is not reached by disturbances arising from the edge of the wedge. This figure corresponds to a case between those of figures 7(*b*) and 7(*c*) previously presented by Skews (1997), which should be consulted for a more comprehensive treatment of the geometry of these edge flows. The current diagram is an oblique perspective view showing one-quarter of the flow field surrounding two wedges in a supersonic wind tunnel. Such a flow has both vertical and horizontal symmetry. In this representation the tunnel centreline is along UV, with the plane UVWTU being in the horizontal plane of symmetry and UVJBU being in the vertical plane of symmetry. This latter plane bisects the lower wedge. AB is the leading edge of the wedge, with a length corresponding to the wedge semi-span, and KL is the wedge trailing edge. The bow wave arising from this leading edge is shown in orange and is made up of two parts: a plane surface ABDEA positioned at a fixed angle to the wedge surface, and which is well predicted by oblique shock theory, and a curved part AECMIA, which diffracts around the wedge edge. Disturbances arising from the edge propagate at sound speed into the uniform flow between this plane oblique shock and the wedge surface. The front of these disturbances will be on the surface of the Mach cone associated with the Mach number of this flow. The segment of the Mach cone between the wedge and bow shock is the reddish surface AFEA with the edge AF on the surface of the wedge being at the Mach angle of this flow to the edge AK. The incident bow wave reflects from the horizontal plane of symmetry generating the reflected wave DECMRD, shown in green. The upstream Mach cone intersects the reflected wave along EF. The perturbations carried with it then propagate into the flow region behind the reflected shock along a cyan coloured Mach cone front EGHE. Each point along EF will radiate perturbations arising from the wedge edge into the flow behind the reflected shock. In the case shown the signals arising from E run ahead of those from F, although this is not always the case, and the edge signal will influence the reflected shock along the line EG. This means that only the portion DEGRD of the reflected shock remains plane and unaffected by edge effects. It is interesting to note that over the area EFGE the conditions behind the reflected shock are changed by being exposed to the secondary Mach cone EGHE even though the upstream Mach cone has not yet influenced this area. It may be expected that the shape of the reflected shock would then be modified over this area.

This feature becomes of major importance to the issue of wind tunnel testing to find the transition angle. When the flow behind the reflected shock approaches the sonic condition the Mach angle of the cone EGH will sweep forward toward the line of reflection ED, until at the sonic condition the whole of the back side of the reflected shock becomes accessible to edge perturbations.

At some point C there will be a transition from regular reflection along the curved path EC to Mach reflection further out. A growing Mach stem surface CNMC (shown in blue) then results. A slipstream surface CMSC (coloured magenta) is generated from the triple line CM and is convected downstream. The direction of the streamline CS is outwards due to the direction of the pressure gradient resulting from the expanding flow around the wedge edge.

Point C is of major interest. A streamline immediately on one side of it passes through a regular reflection whereas that on the other side passes through a Mach stem. This condition of a regular reflection in equilibrium with a Mach reflection has been termed the mechanical equilibrium point (Henderson & Lozzi 1975) or von Neumann point. However, up until now this concept has been used in the context of shock reflection transition in two-dimensional flows where, as the wedge angle increases from the regular reflection case, this is the first stage where Mach reflection becomes possible. Thus the two reflection types are separated in time. In the current case, however, the regular and Mach reflection co-exist adjacent to each other at the same point and at the same time, with equal pressure rise. It is suggested (Skews 1997) that it would not be possible to have reflection transition at any other condition than mechanical equilibrium as point C moves towards the two-dimensional flow at E as the wedge angle increases, since there is no mechanism to sustain a strong pressure difference for two-dimensional regular reflection above point Q on the polar (figure 1) and an adjacent Mach reflection at Q. Thus if in figure 2 the two-dimensional regular reflection BDR was just at the von Neumann point then point C of the peripheral Mach reflection and point E of the two-dimensional regular reflection would coincide. Any increase of the wedge angle into the dual solution domain would result in a higher pressure behind the regular reflection which cannot be matched by an equivalent increase on the Mach reflection side of point E, as is evident from figure 1. It is suggested that this pressure mismatch could act as the trigger to cause the reflection along DE to transit to Mach reflection. The rate at which this would occur is not known. This phenomenon could be a contributory cause (other than noise) for all high aspect ratio experimental tests showing transition in the vicinity of the von Neumann point. If so, the experimental findings could be replicated in an ideal, noise-free wind tunnel. High-resolution three-dimensional simulations may help to resolve this hypothesis. It should be mentioned that the conventional terminology for two-dimensional flows is being applied to this three-dimensional flow case in order to convey the same special or limiting meaning (such as von Neumann or detachment), but the actual values of wedge angles for these cases, and indeed the theory to predict them, are not those for the two-dimensional von Neumann case.

The above considerations do not preclude hysteresis in three-dimensional flows. In fact it is to be expected. It is clearly possible for regular reflection on the centreline under conditions above Q on the polar (figure 1), with conditions changing gradually in the transverse direction along the curved reflection line with reducing downstream pressure, and then through the von Neumann condition where the regular reflection changes to Mach. This model would be entirely consistent with the undulating Mach surface found by Ivanov *et al.* (1997*a*) in their numerical simulations, where the reflection on the centreline could exceed detachment conditions and thus would have to transit to Mach reflection, but the transverse flow field further out would allow regular reflection which would then in turn be encroached on by the peripheral Mach reflection.

3. Apparatus

The current study is aimed at exploring the flow geometry in truly three-dimensional flows between two wedges and in particular that during transition from regular reflection to Mach reflection. For this reason the wedges are made narrower than in previous studies. The wedges themselves have a span of 20 mm and a chord of 40 mm, i.e. a geometrical aspect ratio of 0.5. Replaceable wedges of different angles are fitted in pairs to a forked sting, which can be placed at two orthogonal positions in the tunnel, so that both plan and elevation views of the flow can be obtained. The wedge mounting system is such that the inlet aspect ratio (wedge span divided by distance between leading edges) changes very slightly for different wedge angles. This ratio is 0.56 near transition conditions on the tunnel centreline and thus is well within the region which ensures that the reflection point is exposed to the edge expansion waves (Skews 1997), resulting in curved incident shocks and a fully three-dimensional reflection pattern. Thus these tests do not specifically test for hysteresis since the wedges are static, but are rather aimed at exploring techniques which will elucidate the flow features, and which can subsequently be extended to dynamic testing.

The wind tunnel is of the blowdown type with the Mach number variation achieved with a sliding nozzle block. The test section cross-section is 100×100 mm. All the tests described are run at a Mach number of 3.1, which is towards the upper limit of the operating range. Higher Mach numbers would have been desirable in order to test under conditions where there is a larger difference between detachment and von Neumann conditions (22.02° and 19.90° for $M = 3.1$).

The contact shadowgraph system consists of a Xenon light source with an exposure time of 1 μs situated at the focal distance of a parabolic mirror, thus resulting in a parallel beam of light. A short-duration light source was chosen to minimize any image smearing that might result from noise or vibrations in the tunnel. This optical assembly is then rotated to the required optical yaw angle with the constant-diameter light beam passing through the required portion of the flow field. The optical yaw angles quoted in this work are nominal values ($\pm 2°$). Immediately on the other side of the tunnel this beam is intercepted by a film holder containing standard 120 roll film, positioned perpendicular to the light beam. There are no focusing optics and direct full-scale shadow images are recorded on the film. This method is found to give exceptionally high levels of detail, and is superior to conventional schlieren and colour schlieren that have commonly been applied by workers in the field in the past. Comparisons with the latter techniques show that many of the features which will be described below are completely obscured by the grey-scale or colour gradients resulting from the lens-like shape of the density field around the wedges. The oblique

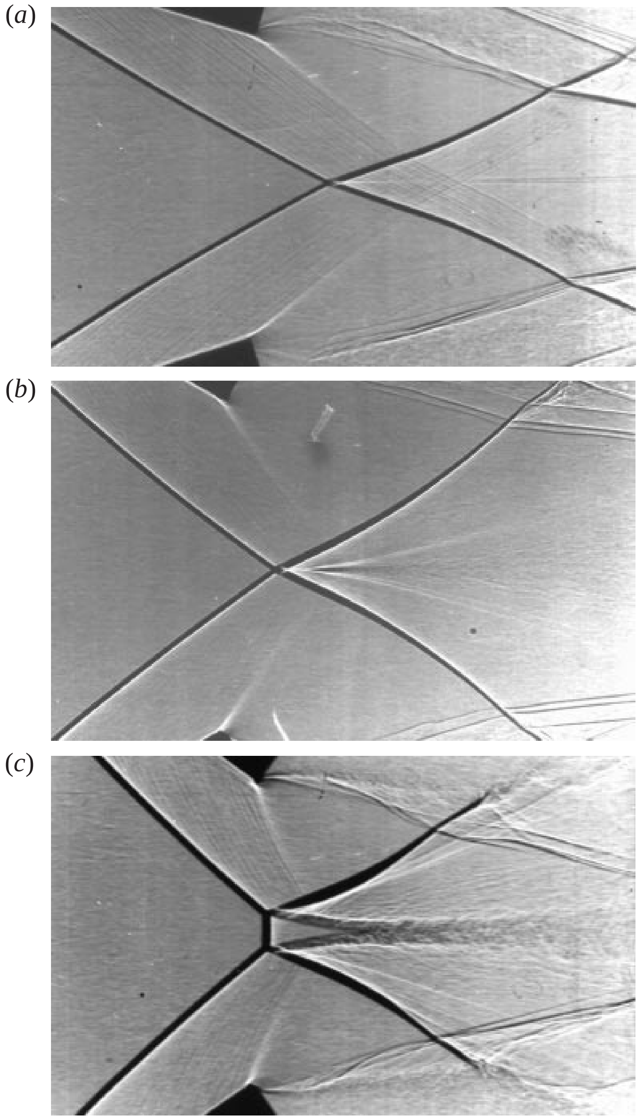

FIGURE 3. Conventional shadowgraph images of three-dimensional flows: (*a*) wedge angle $\delta = 20^\circ$, (*b*) 28°, (*c*) 35°.

shadow images are less easy to interpret than views taken with the conventional view normal to the main flow because they do not represent the familiar view of the phenomena, but they do allow many facets of the three-dimensional flow, which would otherwise be obscured, to be identified.

4. Three-dimensional flow features

Figure 3 shows conventional contact shadowgraphs, i.e. with the optical axis perpendicular to the wind tunnel windows. The exceptionally high fidelity of the image is immediately evident, as noted from the Mach lines emanating from the wedge surface, which is machined (milled). Figure 3(*a*) (wedge angle $\delta = 20^\circ$) shows a familiar image of regular reflection, and figure 3(*c*) ($\delta = 35^\circ$) for a Mach reflection. Such images can, and have in the past, been incorrectly interpreted as two-dimensional flows. However, both show features that give a clue to the essentially three-dimensional nature of the interaction. In the regular reflection case it is a slight horizontal line downstream of the reflection point which theoretically should not exist in a strictly two-dimensional flow if the two incident waves are of identical strength. Furthermore this line appears to split into two lines in close proximity to each other further downstream, which in a two-dimensional flow could be interpreted as a thickening of a slipstream or even that the reflection was Mach reflection and these lines arise from the slipstreams. As will be illustrated later these are shadows of flow features arising from three-dimensional effects. In the Mach reflection case (figure 3*c*) it is the triangular patch immediately downstream of the Mach stem that indicates non-two-dimensionality. For a two-dimensional flow the apex of the triangle, on the plane of symmetry, would not be closed since the gas passing through the Mach stem has to pass between the two slipstreams, which will thus remain separated until mixing causes their merging. In the three-dimensional case, however, this flow will move transversely away from the centre of the flow as the slipstreams make contact on the horizontal plane of symmetry.

Figure 3(*b*) shows a wake system, for a wedge angle between that of figures 3(*a*) and 3(*c*), which is totally different from any expected two-dimensional flow, and as far as is known has not been identified or commented on before. Although the wave reflection itself is evidently regular, a series of lines downstream of the reflection point and straddling a dark patch show evidence of what appears to be significant flow divergence. Further examination of the Mach reflection image of figure 3(*c*) shows further evidence of these lines. An initial interpretation of the cause of these features has been given by Skews *et al.* (1996), based on rather poor quality video images of flow between two moving wedges. In that paper the wedge aspect ratio was larger and it was assumed that the central flow between the wedges was two-dimensional.

It has been found useful in this study to generate three-dimensional models of the expected flow field using a suitable CAD (Computer Aided Design) package. The virtual model can then be viewed from any angle on the monitor and the correspondence with the photographs of the real flow examined. This makes it relatively easy to identify particular features in the flow, as to what they are and where they arise. Based on the edge influences given in the previous schematics of the flow (Skews 1997) the fully three-dimensional flow patterns would be as given in figure 4 (page 90), where the expected waves and surfaces are shown within a rectangular box representing the test section of a wind tunnel. The benefits of being able to view surfaces with a degree of transparency is also evident. Two cases are considered, one with regular reflection in the vertical plane of symmetry (as would be seen by a conventional shadow or schlieren system) and one with a Mach reflection in this plane. In the first case, as the intersection between the two bow waves sweeps backwards and sideways, so the angle between the waves increases and the reflection will change to Mach reflection. Note that a simpler version of the flow is shown than that predicted from Ivanov's calculations, partly in the interests of simplicity and ease of explanation, and partly because of the experimental results given later. It is, however, an interesting exercise to speculate on the effects of the undulating Mach stem surfaces he predicts, particularly in terms of the behaviour of the contact surfaces that they shed. Whilst the current geometrical model is schematic, consideration is being given to developing a true scale model of the wind tunnel flow so that finer details of the interaction can be interpreted.

In figure 4 the plane TUVW is the horizontal plane of symmetry between the two wedges. TU represents the width of the wind tunnel between the windows. The

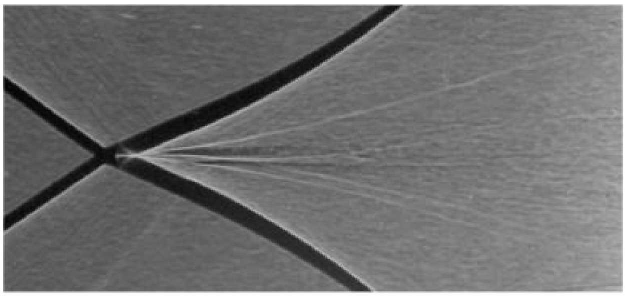

FIGURE 5. Effect of optical yaw and roll on the image for flow corresponding to figure 3(*b*).

lower-wedge leading edge is at AB. The upper wedge and wave systems are not shown in the interests of clarity, but are simply mirror images in plane TUVW of those below. Wave surfaces reflected off the side, top, and bottom of the tunnel, and the wakes behind the wedge are not shown.

The incident wave ABIMCDC′M′I′A is shown in orange in figure 4(*a*), and the reflected wave RMCDC′M′R′R in green. The blue surfaces MNCM and M′N′C′M′ are the two peripheral Mach stem surfaces arising from the bow wave interaction away from the body, as described before, with the lines MC and M′C′ being triple lines (locus of the triple points in a three-dimensional Mach reflection). It is at C and C′ that the reflection changes from regular to Mach. Since the flow through the Mach stem surface results in a different thermodynamic state from that through the adjacent incident and reflected wave surfaces, a slipstream surface CMQSC, coloured magenta, will be shed from the triple line. A typical Mach reflection pattern is evident where this wave system meets the tunnel window surface, with IM the incident wave, RM the reflected wave, MN the Mach stem, and MQ the slipstream.

The second schematic, shown in figure 4(*b*), represents the expected wave pattern when the Mach stem surface has propagated all the way to the centre of the tunnel and the reflection between the two bow waves is of Mach format everywhere. The slipstream sheet will then also extend across the full width of the wind tunnel. As noted previously for figure 3(*c*) the lower edge of the slipstream emanating from the triple point in the vertical plane of symmetry (point D in figure 4*b*) joins the one generated from the upper wedge. To represent this, the slipstream sheet shown in figure 4(*b*) is drawn in contact with the horizontal mid-plane at S. This would suggest that the flow passing through the Mach shock MNCD must move transversely away from the tunnel centre to exit through the open gap QVS. In reality the slipstream is a finite-thickness mixing layer and thus some of the slower flow that has passed through the Mach stem will be entrained into this layer. Nevertheless there will be a significant transverse component of velocity driven by the pressure gradient resulting from the shape of the Mach stem. This is evident in the numerical calculations of Ivanov *et al.* (1997*b*) and in the significantly decreasing length of the clear triangular patch immediately behind the Mach stem, relative to the Mach stem height, as the aspect ratio decreases in experiments.

5. Oblique photographic results and discussion

That the schematic given in figure 4(*a*), with regular reflection on the centreline and Mach reflection further out represents the actual flow, is confirmed by slightly tilting the optical axis. Figure 5 shows a flow under similar conditions to that of 3(*b*) but

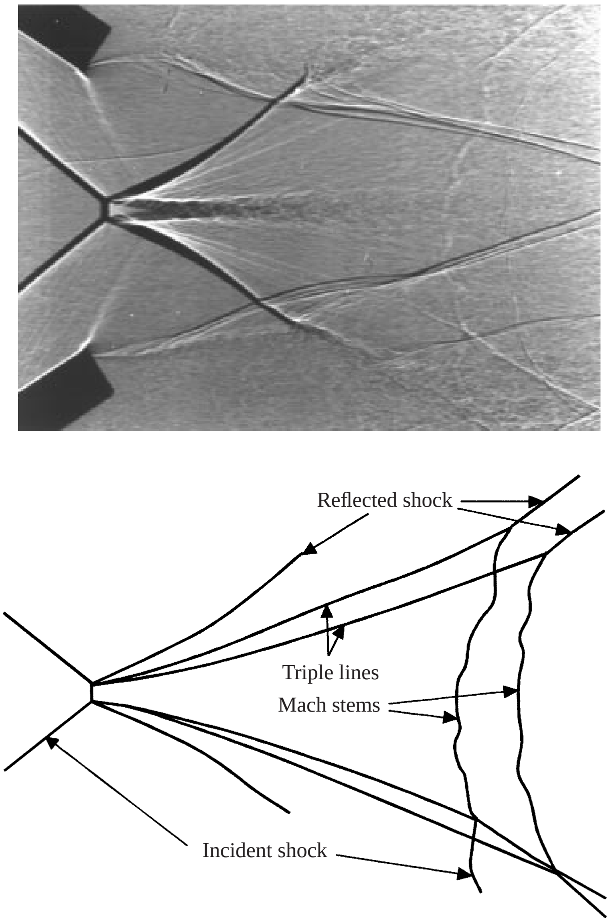

FIGURE 6. Shadowgraph and tracing, showing the interaction of the bow wave system with the wind tunnel window.

with an optical axis with approximately two degrees of yaw and two degrees of roll relative to the wind tunnel axis, and a wedge angle of 24.5°. The apparent diverging wake of figure 3(*b*) then resolves into two separate features characterized by two dark areas rather than one. The start of each of these dark areas occurs where a single white line emanating from the point of reflection between the incident shock waves splits into a pair of white lines with the dark patch between them. Comparison with the schematic leads to the following interpretation. The transition between the single white line and the double line corresponds to the point of transition between regular and Mach reflection i.e. points C and C′ in the schematic. Thus the single line from the start of the one dark patch through the point of reflection of the shocks and back to the start of the other patch is simply the shadow of the line of regular reflection C′DC . The white lines on either side of each of the dark patches are the shadows of the triple line CM (and C′M′) and their (mirror-image) companion lines in the upper half of the flow. The increasing distance between these pairs of lines thus gives a direct measure of how the Mach stem height grows from zero at C to MN at the tunnel wall. The dark patches are themselves shadows of the curved slipstream surface.

The shadows of the triple lines are also clearly visible in cases where Mach reflection

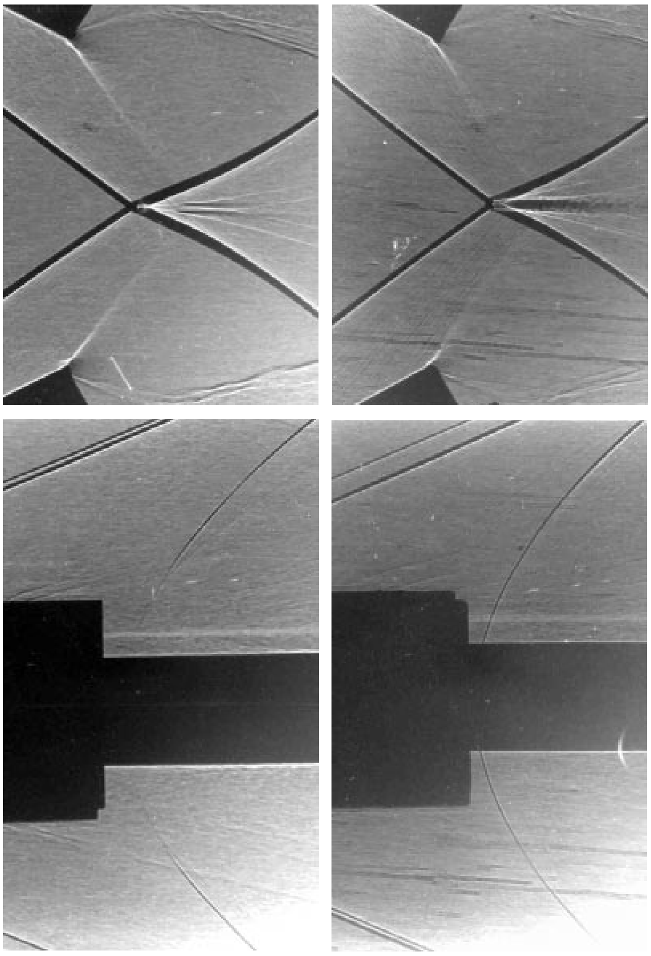

Figure 7. Top and normal views of orthogonal shadowgraphs with regular (left) and Mach reflection (right) on the tunnel centreline.

occurs across the whole tunnel width, and are clearly evident in figure 3(*c*) where they have diverged to such an extent due to the substantial growth in the height of the stem that they are close to the shadow of the reflected wave. Tilting the optical axis by a few degrees separates the shadows of the triple lines existing on either side of the flow, and furthermore, if the flow further downstream of the wedge is examined the interaction of the whole shock system with the windows can be seen. Thus in figure 6 the shadows of the stem, reflected shock and triple lines on both windows are evident, corresponding to lines MN, MI, and MD in figure 4, with the stem images offset from each other because of the optical tilt. The Mach stem has a distinct convex shape facing towards the oncoming flow, which corresponds to the numerical findings of Ivanov *et al.* (1997*a*). The unevenness of the stems may possibly be ascribed to the fact that the reflection on the window is within a boundary layer flow. As will be noted later, this interaction with the window surface sometimes shows up as multiple lines which could well relate to the complex nature of this boundary layer interaction.

The supporting fork for the wedges can be rotated by 90°, which enables top views of the interaction to be obtained. Two such results are given in figure 7, together with the corresponding normal views. The narrower black shadow extending to the right is that of the sting support. Slight optical offsets are apparent in both cases. For the top view this is evident from the slight offset between the trailing edges of the two wedges, and the lack of coincidence between the two bow shocks visible in the left top and bottom corners of these pictures. The top views are helpful in that they establish the shape of the reflection line in the horizontal plane of symmetry (Curves NCDC′N′ and NCN′ in figure 4) as well as looking directly down on the Mach stem surfaces. The two left-hand images of figure 7 correspond to the case with regular reflection on the tunnel centreline. This is evident from the side view where the two transition points between regular and Mach reflection are clearly visible, as described above for figure 4. In the orthogonal view the shadow of the peripheral Mach surfaces shows up as a dark line up to the corresponding position where transition occurs, and then changes to a much lighter line passing under the shadow of the sting, corresponding to the line of regular reflection. Of particular interest is the slightly darker area arising from the transition point (particularly visible in the upper half of this image) and curving away behind the Mach stem line. The only feature that this could correspond to in figure 4 is the edge of the contact surface CS; however, the outward motion of the edge of this region appears excessive for this to be so, and additional investigation is required. The two images on the right in figure 7 correspond to the case where Mach reflection occurs throughout the flow ($\delta = 29°$). The dark line of this Mach stem surface then passes between the model supports. This plan view of the line of reflection does not sweep back by as much as for the regular reflection case, as may be expected because of the blockage caused by the converging contact surfaces at the centre of the flow field.

In view of the useful information obtained from oblique imaging and the clear benefits that could be obtained for the study of transition by viewing it from an even greater angle, a series of tests were conducted with optical yaw angles between 45° and 55°. These give exceptional insight into the flow, as well as the ability to directly image the shape of the Mach stem surface. All that is required experimentally is a wind tunnel with wide windows in the direction of flow to accommodate the inclined beam. There is considerable refraction through the window glass which needs to be accounted for, and more light is needed because of the amount lost through oblique reflection off the window surfaces.

Figure 8 shows a view of the interaction with a 45° angle of yaw of the optical axis using a 27° wedge. The parallel trailing edges of the wedges are evident at the top and bottom of the image. For this case the reflection is regular on the tunnel centreline but the angular optical offset is so large that the reflection between the shadows of the incident waves at the left of the picture shows as a Mach reflection. Referring to figure 4 this is because the light passing through the test section is so angled that it is grazing the peripheral Mach stem surface C′M′N′ rather than the reflection line C′DC. The view orientation is, in fact, similar to that of figure 4 itself with the Mach stem surface foreshortened, except that the optical axis is in the horizontal plane of symmetry. What is of major interest, however, is the small dark arrow-shaped projection immediately behind the reflection point. This is the front part of the Mach stem surface wrapping around between the incident bow waves towards the film plane. The point of the arrow corresponds to point C′ where the reflection changes from Mach to regular, with the line of regular reflection showing as the white horizontal line to the right in the image. This perspective of the Mach stem surface in the vicinity of the point of transition is highly foreshortened, which emphasizes a strong taper towards the tip. This image also shows the shadow of the

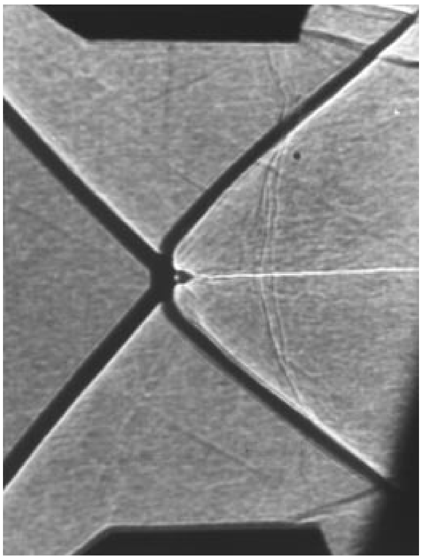

FIGURE 8. Typical features evident in an oblique shadowgraph (triple line, reflection transition point, and window interaction).

Mach stem surface where it strikes the window. This is the slightly wavy curved line between the reflected waves. As noted previously it probably shows up as a double line because of the shock/boundary-layer interaction. The shadow of the reflected shock line on the window (corresponding to M′R′) lies almost directly behind the reflected shock image, as does the shadow of the triple line. The image of the incident shock/window interaction can also be made out between the upper reflected shock and wedge surface.

The evolution of the reflection pattern from mixed regular/Mach reflection to full Mach reflection is shown in figure 9, for a range of wedge angles. The top image is for a 55° optical yaw, 26.5° wedge, with a wider optical view than the previous images. This now shows the encroachment of the second mechanical equilibrium point from the right (point C in figure 4). With reference to this figure it is also apparent that this transition point is being viewed almost head-on, in contrast to the other transition point, C′. A similar image for a 45° optical axis and 27° wedge and closer to complete transition is given in figure 9(*b*), clearly showing the mutual approach of the two peripheral stem surfaces. The right-hand Mach stem surface increases smoothly in height away from the centreline but the details of the transition itself, to regular reflection, is unclear. The two white lines straddling the stem surface on one side of the transition point do not merge smoothly into the single line of the regular reflection. Again there is evidence of a fairly sudden increase from the regular reflection to a finite stem height Mach reflection, as was noted for the left-hand transition. Further studies with improved imaging will be required to establish the exact nature of the stem growth. Increasing the wedge angle by 1° results in Mach reflection throughout as shown in figure 9(*c*), which already shows a significant stem width at the tunnel centreline (situated at about 40% of the image width from the left edge of the image). Interestingly there is a slight indication of the stem being slightly wider at the centre, but this is ambiguous for all images taken, and in view of the marked effects in Ivanov *et al.*'s numerical results at Mach 5.0, indicates that the stem undulations may be sensitive to flow Mach number or inlet aspect ratio. A

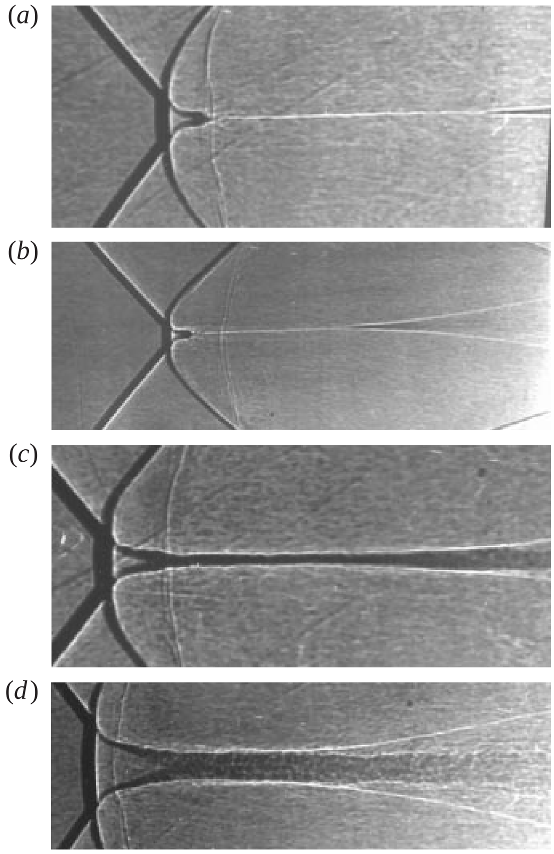

Figure 9. Evolution of the reflection pattern with increasing wedge angle: (*a*) 55° optical yaw angle, 27° wedge angle; (*b*) 45° yaw, 28° wedge; (*c*) 55° yaw, 29° wedge; (*d*) 55° yaw, 32° wedge.

further 2° increase in wedge angle results in a significant increase in the stem width (figure 9*d*). The right-hand side of this image shows a separation of the white triple lines from the dark patch between them, which is the shadow of the main core of the contact surfaces convecting downstream, as noted also in figure 3(*c*).

In some images at the highest wedge angles where regular reflection still exists there are distinct signs of unevenness or instability in the transition region. Figure 10 is a typical example. This image is taken with a 27° wedge and an optical yaw angle where the view is almost tangent to the Mach stem where it meets the wall as is seen by the closeness of the shadow on the wall to the dark shadow of the stem. As the stem wraps forward towards the film plane there is the section near the transition point, as noted previously, of a slowly changing stem height followed by a fairly rapid reduction in height, and then what appears to be a recovery in width before the stem collapses into regular reflection. This type of behaviour is entirely compatible with suggestions of flow unsteadiness triggering the transition. For a less curved reflection line resulting from a higher aspect ratio wedge, the effect could well be to trigger the transition across the full width of the flow. Even in the present case this appears to occur.

Figure 11 is an image taken under the same nominal test condition as figure 10 from a separate test; however, here there is Mach reflection throughout. This image is of interest for a number of other reasons. Firstly the change in Mach stem height is not

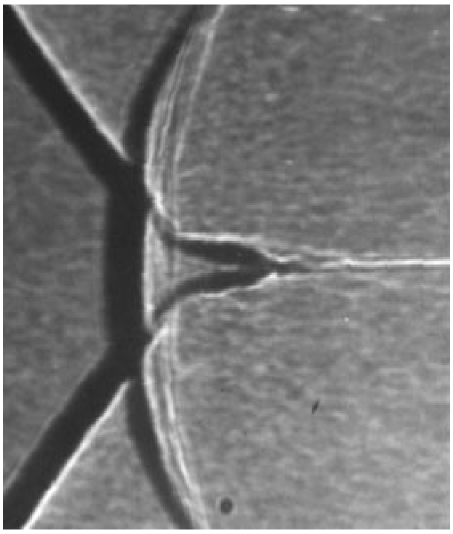

FIGURE 10. Evidence of instability at the transition point.

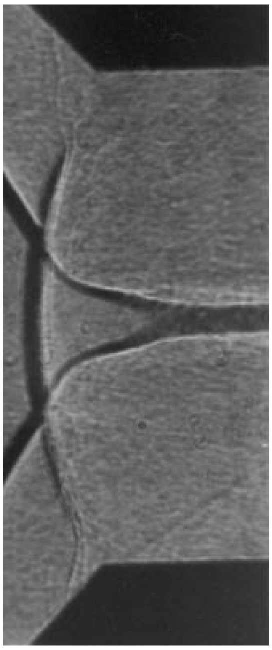

FIGURE 11. View with highly oblique optical axis.

the smooth change evident in figure 9(*d*), but still shows undulations corresponding to the changes in height noted earlier, but now merging with the smoother profile of the stem towards the tunnel centre. Furthermore, before the rapid thinning occurs there are signs of the edges of the contact surface being shed from the thicker stem surface. This effect can confuse the interpretation of the instabilities noted earlier. The second point is that in this image the optical yaw is such that the Mach stem and its interaction with the wall are coincident. As it folds away behind the incident shock surface it gets longer until it meets the wall. The issue of interest is that there is no shadow of the reflected shock, the angle of optical yaw being so large that the light beam is nowhere tangent to its surface. There is a faint record of the incident shock interacting with the wall, and an even fainter, almost invisible trace of the reflected shock interaction with the boundary layer. What is very clear, however, is the convex shape of the Mach stem, confirming the numerical predictions of Ivanov *et al.* (1997*b*). The final point of interest is how quickly the contact surfaces merge, even at sections of the flow more remote from the centreline of the tunnel. In this view the Mach stem surface visualized is positioned well towards the wall. The implication of this is that the area over which the two contact surfaces are merged is substantial.

6. Conclusions

From these tests it is estimated that the wedge angle at which transition occurs is approximately 27°. This is very much higher than the range covered by the two-dimensional dual solution domain, from 20° (von Neumann) to 22° (detachment) for this Mach number. This comparison illustrates very clearly that the two-dimensional results cannot be used for describing these three-dimensional flows. Because of this major influence, it also suggests that apparently two-dimensional flows which are perturbed to a small extent by three-dimensional effects could have very different transition and hysteresis behaviours than the theoretical expectation. It is clear that transverse disturbances due to edge effects will penetrate into the flow field when conducting transition tests because once subsonic conditions are established behind the reflected wave an information path opens up enabling transverse perturbations to penetrate to the rear of the reflected wave. It is suggested that this is a reason why experiments, even using wide wedges in order to avoid three-dimensional effects, do not produce the ideal Hornung hysteresis. Furthermore it is indicated that when three-dimensional effects are significant and the reflection is curved along its full length, that hysteresis may be expected.

The indications are that the phenomenon of transition across to the centre of the flow could be very sensitive to wedge angle as this limiting condition is approached. In the current experiments both Mach and regular reflection results were obtained for the same nominal initial settings near this condition. It is evident that further study on this issue and on hysteresis phenomena in these three-dimensional flows will require dynamic testing with a model that can be moved and photographed throughout a single test. The use of the oblique shadowgraph flow visualization technique as applied in this work will allow unambiguous conditions of transition and hysteresis phenomena to be explored when used in conjunction with dynamic wedge movement. Such tests are currently being planned. In addition, as suggested by a reviewer of this work, it could be informative to relate transition conditions to shock wave angle at the reflection point rather than with wedge angle, since the waves are curved in these three-dimensional flow cases, to establish whether these data correlate with the two-dimensional theories.

The current work has also given indications of the effects that flow unsteadiness and wind tunnel noise may have on the transition phenomena, in that definite signs of instabilities in the flow have been noted in the region of the von Neumann point.

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REFERENCES

CHPOUN, A. & BEN-DOR, G. 1995 Numerical confirmation of the hysteresis phenomenon in the regular to the Mach reflection transition in steady flows. *Shock Waves* **5**, 199–203.

CHPOUN, A., PASSEREL, D., LI, H. & BEN-DOR, G. 1995 Reconsideration of oblique shock wave reflections in steady flows. Part 1. Experimental investigation. *J. Fluid Mech.* **301**, 19–35.

FOMIN, V. M., IVANOV, M. S., KHARITONOV, A. M., KLEMENKOV, G. P., KUDRYAVTSEV, A. N. & HORNUNG, H. G 1996 The study of transition between regular and Mach reflection of shock waves in different wind tunnels. In *12th Intl Mach Reflection Symp.* (ed. B. W. Skews). Book of abstracts and overheads, pp. 138–151.

HENDERSON, L. F. & LOZZI, A. 1975 Experiments on transition of Mach reflection. *J. Fluid Mech.* **68**, 139–155.

HENDERSON, L. F. & LOZZI, A. 1979 Further experiments on transition to Mach reflection. *J. Fluid Mech.* **94**, 541–559.

HORNUNG, H. G. 1997 On the stability of steady-flow regular and Mach reflection. *Shock Waves* **7**, 123–125.

HORNUNG, H. G. & KYCHAKOFF, G. 1977 Transition from regular to Mach reflection of shock waves in relaxing gases. In *Proc. 11th Intl Symp. on Shock Tubes and Waves* (ed. B. Ahlborn, A. Hertzberg & D. Russell), pp. 296–302. University of Washington Press.

HORNUNG, H. G., OERTEL, H. & SANDEMAN, R. J. 1979 Transition to Mach reflection of shock waves in steady and pseudosteady flow with and without relaxation. *J. Fluid Mech.* **90**, 541–559.

HORNUNG, H. G. & ROBINSON, M. L. 1982 Transition from regular to Mach reflection of shock waves, Part 2. The steady flow criterion. *J. Fluid Mech.* **123**, 155–164.

IVANOV, M. S., GIMELSHEIN, S. F. & BEYLICH, A. E. 1995 Hysteresis effect in stationary reflection of shock waves. *Phys. Fluids* **7**, 685–687.

IVANOV, M., GIMELSHEIN, S. F., KUDRYAVTSEV, A. N. & MARKELOV, G. N. 1997*a* Transition from regular to Mach reflection in two- and three-dimensional flows. In *Proc. 21st Intl Symp. on Shock Waves* (ed. A. F. P. Houwing & A. Paull), pp. 813–818. Panther Publishing.

IVANOV, M., KLEMENKOV, G. P., KUDRYAVTSEV, A. N., NIKIFOROV, S. B., PAVLOV, A. A., FOMIN, V. M., KHARITONOV, A. M., KHOTYANOVSKY, D. V. & HORNUNG, H. G. 1997*b* Experimental and numerical study of the transition between regular and Mach reflections of shock waves in steady flows. In *Proc. 21st Intl Symp. on Shock Waves* (ed. A. F. P. Houwing & A. Paull), pp. 819–824. Panther Publishing.

IVANOV, M., ZEITOUN, D., VUILLON, J., GIMELSHEIN, S. & MARKELOV, G. 1996 Investigation of the hysteresis phenomena in steady shock reflection using kinetic and continuum methods. *Shock Waves* **5**, 341–346.

MARCONI, F. 1983 Shock reflection transition in three-dimensional steady flow about interfering bodies. *AIAA J.* **21**, 707–713.

NEUMANN, J. VON. 1943 Oblique reflection of shocks. *Explos. Res. Rep.*, vol. 12. Navy Dept.

SCOTT, D. M. & SKEWS, B. W. 1996 Shock wave reflection in steady flow. *Aeronautica Meridiana* **12**, 36–42.

SKEWS, B. W. 1997 Aspect ratio effects in wind tunnel studies of shock wave reflection transition. *Shock Waves* **7**, 373–383.

SKEWS, B. W., VUKOVIC, S. & DRAXL, M. 1996 Three-dimensional effects in steady flow shock wave reflection transition. In *12th Intl Mach Reflection Symp.* (ed. B. W. Skews). Book of abstracts and overheads, pp. 138–151.